STUDENT PROJECT

ON

An overview of the History of Mathematics

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An overview of the History of Mathematics

Mathematics starts with counting. It is not reasonable, however, to suggest that early counting was mathematics. Only when some record of the counting was kept and, therefore, some representation of numbers occurred can mathematics be said to have started.

In Babylonia mathematics developed from 2000 BC. Earlier a place value notation number system had evolved over a lengthy period with a number base of 60. It allowed arbitrarily large numbers and fractions to be represented and so proved to be the foundation of more high powered mathematical development.

Number problems such as that of the Pythagorean triples (a,b,c) with $a^2+b^2 = c^2$ were studied from at least 1700 BC. Systems of linear equations were studied in the context of solving number problems. Quadratic equations were also studied and these examples led to a type of numerical algebra.

Geometric problems relating to similar figures, area and volume were also studied and values obtained for π.

The Babylonian basis of mathematics was inherited by the Greeks and independent development by the Greeks began from around 450 BC. Zeno of Elea's paradoxes led to the atomic theory of Democritus. A more precise formulation of concepts led to the realisation that the rational numbers did not suffice to measure all lengths. A geometric formulation of irrational numbers arose. Studies of area led to a form of integration.

The theory of conic sections shows a high point in pure mathematical study by Apollonius. Further mathematical discoveries were driven by the astronomy, for example the study of trigonometry.

The major Greek progress in mathematics was from 300 BC to 200 AD. After this time progress continued in Islamic countries. Mathematics flourished in particular in Iran, Syria and India. This work did not match the progress made by the Greeks but in addition to the Islamic progress, it did preserve Greek mathematics. From about the 11th Century Adelard of Bath, then later Fibonacci, brought this Islamic mathematics and its knowledge of Greek mathematics back into Europe.

Major progress in mathematics in Europe began again at the beginning of the 16th Century with Pacioli, then Cardan, Tartaglia and Ferrari with the algebraic solution of cubic and quartic equations. Copernicus and Galileo revolutionised the applications of mathematics to the study of the universe.

The progress in algebra had a major psychological effect and enthusiasm for mathematical research, in particular research in algebra, spread from Italy to Stevin in Belgium and Viète in France.

The 17th Century saw Napier, Briggs and others greatly extend the power of mathematics as a calculatory science with his discovery of logarithms. Cavalieri made progress towards the calculus with his infinitesimal methods and Descartes added the power of algebraic methods to geometry.

Progress towards the calculus continued with Fermat, who, together with Pascal, began the mathematical study of probability. However the calculus was to be the topic of most significance to evolve in the 17th Century.

Newton, building on the work of many earlier mathematicians such as his teacher Barrow, developed the calculus into a tool to push forward the study of nature. His work contained a wealth of new discoveries showing the interaction between mathematics, physics and astronomy. Newton's theory of gravitation and his theory of light take us into the 18th Century.

However we must also mention Leibniz, whose much more rigorous approach to the calculus (although still unsatisfactory) was to set the scene for the mathematical work of the 18^{th} Century rather than that of Newton. Leibniz's influence on the various members of the Bernoulli family was important in seeing the calculus grow in power and variety of application.

The most important mathematician of the 18^{th} Century was Euler who, in addition to work in a wide range of mathematical areas, was to invent two new branches, namely the calculus of variations and differential geometry. Euler was also important in pushing forward with research in number theory begun so effectively by Fermat.

Toward the end of the 18^{th} Century, Lagrange was to begin a rigorous theory of functions and of mechanics. The period around the turn of the century saw Laplace's great work on celestial mechanics as well as major progress in synthetic geometry by Monge and Carnot.

The 19^{th} Century saw rapid progress. Fourier's work on heat was of fundamental importance. In geometry Plücker produced fundamental work on analytic geometry and Steiner in synthetic geometry.

Non-euclidean geometry developed by Lobachevsky and Bolyai led to characterisation of geometry by Riemann. Gauss, thought by some to be the greatest mathematician of all time, studied quadratic reciprocity and integer congruences. His work in differential geometry was to revolutionise the topic. He also contributed in a major way to astronomy and magnetism.

The 19^{th} Century saw the work of Galois on equations and his insight into the path that mathematics would follow in studying fundamental operations. Galois' introduction of the group concept was to herald in a new direction for mathematical research which has continued through the 20^{th} Century.

Cauchy, building on the work of Lagrange on functions, began rigorous analysis and began the study of the theory of functions of a complex variable. This work would continue through Weierstrass and Riemann.

Algebraic geometry was carried forward by Cayley whose work on matrices and linear algebra complemented that by Hamilton and Grassmann. The end of the 19th Century saw Cantor invent set theory almost single handedly while his analysis of the concept of number added to the major work of Dedekind and Weierstrass on irrational numbers

Analysis was driven by the requirements of mathematical physics and astronomy. Lie's work on differential equations led to the study of topological groups and differential topology. Maxwell was to revolutionise the application of analysis to mathematical physics. Statistical mechanics was developed by Maxwell, Boltzmann and Gibbs. It led to ergodic theory.

The study of integral equations was driven by the study of electrostatics and potential theory. Fredholm's work led to Hilbert and the development of functional analysis.

Notation and communication

There are many major mathematical discoveries but only those which can be understood by others lead to progress. However, the easy use and understanding of mathematical concepts depends on their notation.

For example, work with numbers is clearly hindered by poor notation. Try multiplying two numbers together in Roman numerals. What is MLXXXIV times MMLLLXIX? Addition of course is a different matter and in this case Roman numerals come into their own, merchants who did most of their arithmetic adding figures were reluctant to give up using Roman numerals.

What are other examples of notational problems. The best known is probably the notation for the calculus used by Leibniz and Newton. Leibniz's notation lead more easily to extending the ideas of the calculus, while Newton's notation although good to describe velocity and acceleration had much less potential when functions of two variables were considered. British mathematicians who patriotically used Newton's notation put themselves at a disadvantage compared with the continental mathematicians who followed Leibniz.

Let us think for a moment how dependent we all are on mathematical notation and convention. Ask any mathematician to solve $ax = b$ and you will be given the answer $x = b/a$. I would be very surprised if you were given the answer $a = b/x$, but why not. We are, often without realising it, using a convention that letters near the end of the alphabet represent unknowns while those near the beginning represent known quantities.

It was not always like this: Harriot used a as his unknown as did others at this time. The convention we use (letters near the end of the alphabet representing unknowns) was introduced by Descartes in 1637. Other conventions have fallen out of favour, such as that due to Viète who used vowels for unknowns and consonants for knowns.

Of course $ax = b$ contains other conventions of notation which we use without noticing them. For example the sign "=" was introduced by Recorde in 1557. Also ax is used to denote the product of a and x, the most efficient notation of all since nothing has to be written!

Brilliant discoveries?

It is quite hard to understand the brilliance of major mathematical discoveries. On the one hand they often appear as isolated flashes of brilliance although in fact they are the culmination of work by many, often less able, mathematicians over a long period.

For example the controversy over whether Newton or Leibniz discovered the calculus first can easily be answered. Neither did since Newton certainly learnt the calculus from his teacher Barrow. Of course I am not suggesting that Barrow should receive the credit for discovering the calculus, I'm merely pointing out that the calculus comes out of a long period of progress starting with Greek mathematics.

Now we are in danger of reducing major mathematical discoveries as no more than the luck of who was working on a topic at "the right time". This too would be completely unfair (although it does go some why to explain why two or more people often discovered something independently around the same time). There is still the flash of genius in the discoveries, often coming from a deeper understanding or seeing the importance of certain ideas more clearly.

How we view history

We view the history of mathematics from our own position of understanding and sophistication. There can be no other way but nevertheless we have to try to appreciate the difference between our viewpoint and that of mathematicians centuries ago. Often the way mathematics is taught today makes it harder to understand the difficulties of the past.

There is no reason why anyone should introduce negative numbers just to be solutions of equations such as $x + 3 = 0$. In fact there is no real reason why negative numbers should be introduced at all. Nobody owned -2 books. We can think of 2 as being some abstract property which every set of 2 objects possesses. This in itself is a deep idea. Adding 2 apples to 3 apples is one matter. Realising that there are abstract properties 2 and 3 which apply to every sets with 2 and 3 elements and that $2 + 3 = 5$ is a general theorem which applies whether they are sets of apples, books or trees moves from counting into the realm of mathematics.

Negative numbers do not have this type of concrete representation on which to build the abstraction. It is not surprising that their introduction came only after a long struggle. An understanding of these difficulties would benefit any teacher trying to teach primary school children. Even the integers, which we take as the most basic concept, have a sophistication which can only be properly understood by examining the historical setting.

A challenge

If you think that mathematical discovery is easy then here is a challenge to make you think. Napier, Briggs and others introduced the world to logarithms nearly 400 years ago. These were used for 350 years as the main tool in arithmetical calculations. An amazing amount of effort was saved using logarithms, how could the heavy calculations necessary in the sciences ever have taken place without logs.

Then the world changed. The pocket calculator appeared. The logarithm remains an important mathematical function but its use in calculating has gone for ever.

Here is the challenge. What will replace the calculator? You might say that this is an unfair question. However let me remind you that Napier invented the basic concepts of a mechanical computer at the same time as logs. The basic ideas that will lead to the replacement of the pocket calculator are almost certainly around us.

We can think of faster calculators, smaller calculators, better calculators but I'm asking for something as different from the calculator as the calculator itself is from log tables. I have an answer to my own question but it would spoil the point of my challenge to say what it is. Think about it and realise how difficult it was to invent non-euclidean geometries, groups, general relativity, set theory, ...

References

General bibliography of about 700 items

Other Web sites:

Astroseti (A Spanish translation of this article)

Article by: *J J O'Connor* and *E F Robertson*